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BCA/M-23 **20576**

MATHEMATICAL FOUNDATION–I

Paper–BCA-105

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question each from Unit-I to Unit-IV. All questions carry equal marks.

Compulsory Question

1. (a) Find the symmetric difference of sets A and B where A = {1, 2, 4, 5} and B = {2, 5, 8, 9}.

(b) Find the differential equation of all parabolas whose axes are parallel to y-axis.

(c) Use tenth tables to show $(p \wedge q) \wedge \sim (p \vee q)$ is a contradiction.

(d) Show that matrix A satisfies the equation $f(x) = 0$ where

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \text{ and } f(x) = x^2 - 5x + 7.$$

(e) Differentiate $\dfrac{x^2}{1+x^2}$ w.r.t. x^2.

UNIT–I

2. (a) How many different words can be formed with the letters of the world 'BHARAT' ?

(i) In how many of these B and H are never together ?

(ii) How many of these begin with B and end with T ?

(b) If $y \log x = x - y$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

3. (a) Examine the continuity of

$$f(x) = \begin{cases} \dfrac{x^{1/x} - 1}{e^{1/x} + 1} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \text{ at } x = 0.$$

(b) Determine whether the set {1, 2, 4, 8, 16} is a lattice with the relation of divisibility.

UNIT–II

4. (a) Solve the differential equation $x\frac{dy}{dx} - y = \sqrt[x]{x^2 + y^2}$.

(b) Solve $(x^2 + y^2 + 2x)\, dx + 2y\, dy = 0$.

5. (a) Solve the differences equation $\frac{d^2y}{dx^2} + a^2 y = \sin ax$.

(b) Solve $(x^2D^2 - 3xD + 5)y = \sin(\log x)$ where

$$D = \frac{d}{dx}, \text{ and } D^2 \frac{d^2}{dx^2}.$$

UNIT–III

6. (a) Prove that $p \Rightarrow (q \wedge r) \equiv (p \Rightarrow q) \wedge (p \Rightarrow r)$ by constructing truth tables, where p, q, r are statements.

(b) Using principle of mathematical modulation to prove that

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots + \frac{1}{x(x+1)} = \frac{n}{n+1}.$$

7. (a) If H_1 and H_2 are two subgroups of G, then H_1 of H_2 is also a subgroup of G.

(b) Define Ring, Subring, Ideal and Field with an example.

UNIT–IV

8. (a) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ then find $\vec{A}$ and show that

$\vec{A} = A^2$.

(b) Reduce the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$ to $[I_3 O]$.

Hence find the rank of A.

9. (a) Find the eigen value of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. Also find the eigen vector corresponds to any one of eigen value.

(b) Prove that eigen values of a Hermitian matrix are all real.
